OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2642

Probability & Statistics 2

Monday **19 JANUARY 2004** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

 Registered Charity Number: 1066969 **[Turn over**

1 The number of currants in a randomly chosen fruit scone can be modelled by a Poisson distribution with mean $4\frac{2}{3}$.

(i) Calculate the probability that, in one randomly chosen fruit scone, there are exactly 3 currants. [2]

(ii) Use Poisson tables to find the probability that, in 3 randomly chosen fruit scones, there is a total of no more than 11 currants. [2]

2 The masses, X grams, of plums on a plum tree can be assumed to be normally distributed with mean μ grams and standard deviation σ grams.

(i) The mean mass in grams of 8 randomly chosen plums is denoted by $\bar{X}$. State the distribution of $\bar{X}$, giving parameters in terms of μ and σ. [1]

(ii) The masses of a random sample of 8 plums are summarised by

$$\Sigma x = 148.0, \qquad \Sigma x^2 = 2809.68.$$

Calculate unbiased estimates of μ and σ^2. [3]

3 A year group in a school contains 200 pupils. A random sample of 8 pupils is to be selected from the year group.

(i) Describe how 3-figure random numbers (obtained from either tables or a calculator) could be used to select the sample. [2]

(ii) 120 of the 200 pupils have surnames beginning with letters in the first half of the alphabet (from A to M). Use a binomial distribution to estimate the probability that at least 6 of the 8 selected in the random sample have surnames beginning with letters in the first half of the alphabet. [3]

4 The random variable X has the distribution $N(10, \sigma^2)$. It is given that $P(X < 7) = p$ and $P(X < 13) = 2p$.

(i) Show that the value of p is $\frac{1}{3}$. [3]

(ii) Find the value of σ. [3]

5 Red lights used for traffic lights have a colour measured by wavelength L, in suitable units. It is known that the random variable L has the distribution $N(\mu, 10^2)$. For those lights in standard use, the value of μ is 660. A random sample of n lights is taken from the output of a new manufacturer. The sample mean wavelength is 657. A test is carried out, at the 5% significance level, of the hypothesis $H_0 : \mu = 660$ as opposed to $H_1 : \mu \neq 660$, for this manufacturer.

(i) The result of the test is to reject H_0. Calculate the smallest possible value of n. [5]

(ii) State the probability of a Type I error occurring as a result of the test. [1]

(iii) Given that a Type I error does occur, state what can be said about the null hypothesis. [1]

6 A hotel has 10 rooms, which are always occupied, and in each room there is a complimentary packet of biscuits. If on any night the packet is opened, it is replaced by a new packet next day. The hotel manager knows from experience that, for each room and for each night, the probability that the packet is opened is 0.4, independently of other rooms and other nights. He keeps spare packets in stock.

(i) The manager needs to be at least 95% sure that he has enough packets in stock each day to be able to replace those that are opened. Find the smallest number of packets that he needs to have in stock each day. [3]

(ii) At the start of a week (7 days), there are 35 packets in stock. Use a suitable approximation to calculate the probability that this is enough to meet the demands in the coming week. [6]

7 Metal struts are designed to be of a given length, but in the manufacturing process the length is subject to an error represented by the continuous random variable X. Two models are proposed for X, with probability density functions as follows.

$$\text{Model 1:} \qquad \mathrm{f}_1(x) = \begin{cases} k_1 & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{Model 2:} \qquad \mathrm{f}_2(x) = \begin{cases} k_2(1-x^2) & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k_1 and k_2 are constants.

(i) Sketch, on the same axes, the graphs of $\mathrm{f}_1(x)$ and $\mathrm{f}_2(x)$. [3]

(ii) For each of the two models, describe in everyday terms how the errors in the lengths vary. [2]

(iii) Write down the value of k_1. [1]

(iv) Calculate the standard deviation, σ, of Model 1. [4]

(v) Without further calculation, state with a reason whether the standard deviation of Model 2 is less than, equal to, or greater than σ. [2]

8 (i) On average I receive 8 letters every weekday. On a randomly chosen Monday I received 2 letters.

(a) Use a Poisson distribution to test, at a significance level as close to 5% as possible, whether this is evidence that I receive fewer than 8 letters on a Monday. State your hypotheses clearly. [5]

(b) Find the actual significance level of this test. [2]

(ii) On Saturdays I also receive an average of 8 letters.

(a) State an assumption needed to model the total number of letters I receive on four consecutive Saturdays by a Poisson distribution. [1]

(b) Use a suitable approximation to the Poisson distribution to calculate the probability that, on four consecutive Saturdays, I receive a total of 25 or fewer letters. [5]

1 $X \sim \text{Po}(4\tfrac{2}{3})$ $\quad$ $\text{p}(X=3) = e^{-\frac{14}{3}} \cdot \dfrac{\left(14/3\right)^3}{3!} = 0\cdot159280... = \mathbf{0\cdot159}$ (3 s.f.) [2]

No. of currants in 3 scones $\quad Y \sim \text{Po}(14)$

$\text{p}(Y \le 11) = 0\cdot2600... = \mathbf{0\cdot260}$ (3 s.f.) [2]

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2 $X \sim N\left(\mu, \sigma^2\right)$ $\quad$ $\boldsymbol{\bar{X} \sim \text{N}\left(\mu, \tfrac{\sigma^2}{8}\right)}$ [1]

unbiased estimates …

$\boldsymbol{\hat{\mu}} = \bar{x} = 148/8 = \mathbf{18\cdot5\ g}$ $\quad$ $\boldsymbol{\hat{\sigma}^2} = \tfrac{8}{7}\left\{2809\cdot68/8 - 18\cdot5^2\right\} = \tfrac{8}{7} \times 8\cdot96 = \mathbf{10\cdot24\ g^2}$ [3]

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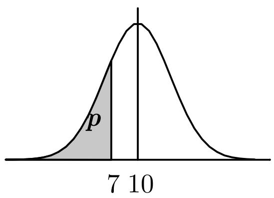
3 **Describe random sampling procedure**.

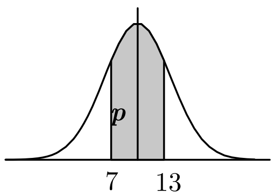
e.g. allocate each pupil a no. from 000 to 199. Pick the sample using random no. generator, ignoring repeats and nos. from 200 to 999. [2]

No. selected with surnames A – M $\quad X \sim \text{B}\left(8,\ 0\cdot6\right)$

$\text{p}(X \ge 6) = {}^8C_6\,(0\cdot6)^6\,(0\cdot4)^2 + {}^8C_7\,(0\cdot6)^7\,(0\cdot4) + (0\cdot6)^8 = 0\cdot315394... = \mathbf{0\cdot315}$ (3 s.f.) [3]

4





$\text{p}(X<7) + \text{p}(7 \le X \le 13) + \text{p}(X>13) = 1$

$p + p + p = 1$

$\boldsymbol{p = \tfrac{1}{3}}$ [3]

$\text{p}(X<13) = \tfrac{2}{3}$ $\quad$ $\Phi\left(\dfrac{13-10}{\sigma}\right) = \tfrac{2}{3}$ $\quad$ $\dfrac{3}{\sigma} = 0\cdot4307$ $\quad$ $\boldsymbol{\sigma = 6\cdot96}$ [3]

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5 On H_0 …. $\quad Z = \dfrac{\bar{X} - 660}{\left(10/\sqrt{n}\right)} \sim \text{N}\left(0,\ 1\right)$ and we reject H_0 when $|Z| > 1\cdot96$

so …. $\quad \dfrac{657-660}{\left(10/\sqrt{n}\right)} < {}^-1\cdot96$ $\quad {}^-0\cdot3\sqrt{n} < {}^-1\cdot96$ $\quad \sqrt{n} > 6\cdot5\dot{3}$ $\quad n > 42\cdot68\dot{4}$

hence the **smallest possible value of $\boldsymbol{n}$ is 43** [5]

$\text{p}(\text{Type I error}) = \mathbf{5\%}$ [1]

If a Type I error does occur (and so H_0 is wrongly rejected) then $\mathbf{H_0}$ **must be valid**. [1]

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6 No. of packets opened each day $\quad X \sim \mathrm{B}\left(10,\ 0\cdot 4\right)$

$\mathrm{p}(X \le \mathrm{N}) \ge 0\cdot 95$ $\quad$ from tables ... $\quad$ $\mathrm{N} \ge 7$

so **the smallest no. of packets that must be kept in stock is 7.**

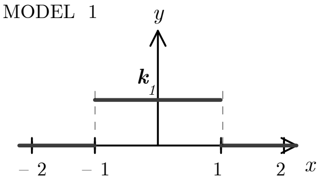
[3]

No. of packets needed in a week $\quad Y \sim \mathrm{B}\left(70,\ 0\cdot 4\right) \approx \mathrm{N}\left(28,\ 16\cdot 8\right)$

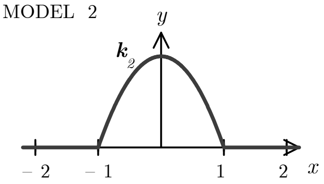
$$\mathrm{p}(Y \le 35) = \Phi\left(\frac{35\cdot 5 - 28}{\sqrt{16\cdot 8}}\right) = \Phi(1\cdot 830) = \mathbf{0\cdot 966}4$$ [6]

7

MODEL 1



MODEL 2



"everyday" descriptions of the two distributions. [3]

[2]

$k_1 = \frac{1}{2}$

[1]

Model 1 $\quad \sigma^2 = \int_{-1}^{+1} \frac{1}{2} x^2 \mathrm{d}x - 0^2 = \left[\frac{1}{6}x^3\right]_{-1}^{+1} = \frac{1}{3}$ $\quad \sigma = \frac{1}{\sqrt{3}} = \mathbf{0\cdot 577}$

[4]

The standard deviation for Model 2 is less than σ as the errors are more likely to be close to zero (the mean).

[2]

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8 $\mathrm{H}_0 : \lambda = 8$ $\quad$ $\mathrm{H}_1 : \lambda < 8$

On H_0 the no. of letters arriving on a Monday $\quad X \sim \mathrm{Po}(8)$ $\quad \begin{cases} \mathrm{p}(X \le 2) = 0\cdot 0138 \\ \mathrm{p}(X \le 3) = 0\cdot 0424 \\ \mathrm{p}(X \le 4) = 0\cdot 0996 \end{cases}$

So with a **4.24% significance level** we reject H_0 if $X \le 3$

For the given Monday, $x = 2$, so that gives us **sufficient evidence on which to reject H_0** and conclude that **I receive fewer than 8 letters on Mondays**.

[7]

one assumption necessary to ensure that total letters over 4 Saturdays is Poisson

[1]

$T \sim \mathrm{Po}(32) \approx \mathrm{N}(32, 32)$

$$\mathrm{p}(T \le 25) = \Phi\left(\frac{25\cdot 5 - 32}{\sqrt{32}}\right) = \Phi\left({}^{-}1\cdot 149\right) = \mathbf{0\cdot 125} \qquad (3 \text{ s.f.})$$

[5]